Financial Modelling of BSE-SENSEX Volatility using ARMA, ARCH and TGARCH Model

Dr. Kahkasha Safi
BBD University,
Lucknow, India
kahkashasefi@gmail.com

Dr. Shraddha Verma
BBD University,
Lucknow, India
Shraddhav19@gmail.com

Abhishek Kumar
Kashi Institute of Technology,
Varanasi, India
abhipriya27@gmail.com

Abstract

Financial decisions taken for the future depend upon the perception of the behaviour of the random variables and the estimation of the variance. The key purpose of the current study is to examine the behaviour and nature of the Indian stock index –SENSEX with the help of GARCH model. The secondary data –SENSEX prices are collected from the official website of BSE for a period ranging from 2011-2020. The result indicates the existence of volatility assembling and persistence. The tail of the series is fatter along the left side and T-GARCH explains that the negative and wicked news affect the stock market more than the good news. We conclude that investors should frame their investment tactics by evaluating the current news in the market and predict the future movements in the market so as to receive maximum possible returns.

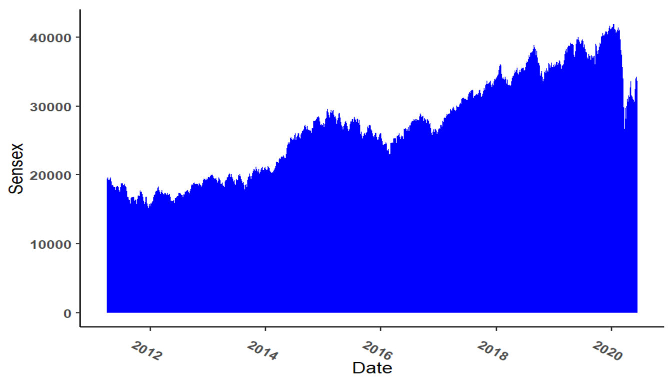
Key words: ARMA; ARCH; T-GARCH; Volatility Modelling; News Impact; India

JEL code: G12, G17

Introduction

Systematic risk is non–diversifiable and is one of the major contributors of risk to portfolios. If the variables defining the systematic risk can be measured, then risk for the financial portfolios is minimised. Volatility is defined as a fluctuation in asset prices and changes in the former is non-random (Tripathy & Rahman, 2013). Hence, financial analysts and econometricians have developed various volatility measuring models. Estimating the volatility is of utmost interest to the various stakeholders like the investors, government, other foreign markets, business etc. Since the interdependence of markets has increased, it is all the more important to predict the dispersions in the stock markets along with measuring the impact of such dispersion on other economies as well. The movement in the stock market is a function of many factors. Therefore, predicting the path of movement is not only difficult, but also not guaranteed. Hence, this study is also an attempt to capture the movement and estimate the volatility of SENSEX, which is one of the valuable stock market indices in India. Especially, given the fact that developing economies like India are the focus of the leading markets, it is all the more important to develop a robust model for presetting the volatility in the markets. Figure 1 depicts the concluding prices of SENSEX.

Figure 1: Closing Prices of Sensex from April 2011-June 2020



This paper is structured as follows - Existing literature related to volatility clustering, sign and size effect on stock indices is reviewed in segment II. Segment III discusses the objectives of the study. Research Methodology and Empirical results are discussed in segment IV. Conclusion is presented in Segment V.

Literature Review

Extensive work is done in the field of forecasting and volatility, for example; Pagan and Schwert (1990) utilized GARCH-type models and compared normal density function models with non-normal ones and some of them favoured skewed Student distributions.

Akgiray (1989) confirmed the validity of the ARCH and GARCH model for predicting S&P 500 Composite Index and Ng and McAleer (2004) used the Nikkei 225 Index as sample to prove the relative effectiveness of GARCH and TGARCH McMillan et al. (2000), studied the UK stock market unpredictability with monthly weekly and daily frequencies and found that daily volatility forecasts were better predicted by the GARCH moving average and exponential smoothing model.

Goyal (2000) selected CRSP value weighted returns (stocks) as sample to investigate the presentation for certain GARCH models. His study reported that predictive ability of the GARCH-M is poorer than a simple ARMA specification. There were some studies, (Banerjee & Sarkar, 2006) and (Pandey, 2005) which supported that E GARCH gives a better result than GARCH. Wilhelmsson (2006) examined the forecasting performance of GARCH(1,1) models using the S&P 500 return series under nine different error distributions. He demonstrated that forecasting results were superior not merely for intraday data in terms of predicting performance when a leptokurtic error distribution was used rather than a normal distribution, but also that these outcomes were valid for daily, weekly, and periodic data. This study decided that despite permitting for deviations in the distribution's complex moments and demonstrating skewness, there were no positive effects on forecast results.

Vasudevan and Vetrivel (2016) supported that the asymmetric GAARCH gives more appropriate results while forecasting the BSE SENSEX returns . There are certain researches which have linked volatility with other factors as well . Garg and Bodla (2011) also used GARCH and took the influence of Foreign institutional investment on the volatility Kumar et al (2016), considered the volatility of option pricing and found GARCH to be a good technique of appropriating it. Laurent, Rombouts, and Violante (2012) having a sample of 10 stocks investigated the best multivariate GARCH models. In this attempt, they conducted 125 diverse GARCH models besides noted that the forecast precision in a 10 years period. They documented that during stock markets instability, multivariate GARCH models produce poor results. Gulay and Emec (2018) compared the predicting performance of the normalization and variance stabilization technique (NoVaS) to that of the GARCH(1,1), EGARCH(1,1), and GJR-GARCH(1,1) models in this article. They investigated the out-of-sample predicting performance of GARCH(1,1)-type models and the NoVaS technique, both of which are based on a generalized error distribution rather than the normal or Student's t-distribution. Results demonstrated that the NoVaS technique outperforms GARCH(1,1)-type models in terms of out-of-sample predicting performance. The result can be used to provide useful guidance when developing models for out-of-sample predicting purposes, with the goal of increasing predicting accuracy.

Bonga (2019) modeled that unpredictability of the Zimbabwean stock market with monthly reappearance sequence comprising of 109 annotations from January 2010 to January 2019. He used both Symmetric and asymmetric specifically: GARCH(1,1), GARCH-M(1,1), IGARCH(1,1) and EGARCH(1,1). The study determines that positive and negative astonishments have differing effects on the series of stock returns. Bad and good news broadcast will upsurge stock market volatility in varying amounts and hence Zimbabwe's investors react to investments differently. Wang, Ma, Liu, and Yang (2020) utilized the GARCH-MIDAS technique for modeling and predicting the stock volatility. Results showed that risky shocks have an important influence on market volatility and that volatility can be partial more by the asymmetry consequence than by the long and short-term risky volatility consequence in their in-sample results, whereas Out-of-sample data showed that their suggested models can obtain improved volatility prediction results. In addition, the improvement in prediction is more firmly related to the incorporation of short-term volatility asymmetry and high volatility consequences.

Objectives

1. To identify if any volatility clustering occurs in the period of study in SENSEX.
2. To identify whether the SENSEX is being affected equally by both positive news and negative news?

Research Methodology & Empirical Results

The study appends the secondary data on daily closing values for BSE-SENSEX 30 for a historical period of 10 years i.e. April 01, 2011- June 17, 2020; this is collected from the official website of BSE. The continuous returns are generated on the daily data of Sensex series, which were computed as follows –

$$RET\_SEN = ln(P)_t - ln(P)_{t-1}$$

Where ln is a natural log to the base e

$ln(P)_t$ Depicts on the day price 't'

$ln(P)_{t-1}$ Depicts one day price before day 't'

The data is collected post global financial crisis of 2008 so as to completely remove its effect from the observations. However, it did include the effect of pandemic as the observations of first quarter of financial year 2020-21 are taken in the study, which helped in studying T-Garch.

Descriptive statistics

Descriptive statistics was found out for return of Sensex series (Refer Table 1) to understand the characteristic of data set. The daily mean return of the series is positive, however, the return varies in the range of -0.141017 to 0.085947. Standard deviation is close to 1%.

Table I: Descriptive statistics for daily returns of Sensex

Statistics	RET_SEN
Sum of Mean	0.0002
Sum of Median	0.0004
Maximum value	0.086
Minimum value	-0.141
Standard Deviation	0.011
Skewness value	-1.138
Kurtosis value	21.505
Jarque-Bera	32604.16
Probability value	0.000
Sum	0.545
Sum Sq. Dev.	0.278
Observations	2251

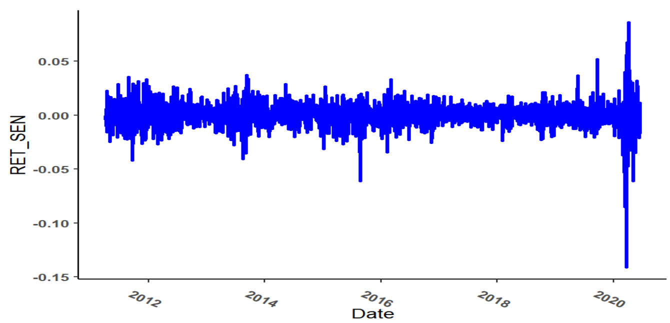
Skewness measures asymmetry of probability distribution of a series around its value of mean. It may be zero, positive and negative. In this case, Skewness is negative which indicates left skewed distribution and the tail is fatter and longer on the left side of the distribution. This can be verified with the fact that the mean value of series is less than the median.

Kurtosis more than 3 indicates that series is not usually distributed and has long right tail. This can be further supported by the rejection of null hypothesis of Jarque Bera test.

Unit Root Test

Since the closing prices of Sensex (refer Figure 1) is not stationary in nature, therefore return series is generated. Unit root test are applied on return series to check their stationarity. The data in time series must be stationary to perform any statistical tools otherwise the spurious results will be obtained. The return series indicate that the value of the mean and their variance of the series is unceasing (Refer Figure 2 and table 1) and does not change with time.

Figure 2: Return Series of Sensex from April 2011-June 2020



Source: The authors'

Table II: Results of Unit root Test

	t-Statistic*
Augmented value of Dickey-Fuller test statistic	-16.452
Phillips-Perron value test statistic	-48.574

*Test critical value with level of significance at 5% is -3.411806; Source: The authors'

The result reported in Table 2 shows that the t-statistic using ADF test is -16.45118 and using PP test is -16.45118, the values in both the cases are much greater than the critical value of -3.411806 with level of significance at 5%. Hence, the return series is stationary.

ARMA Model

Before applying the ARCH and GARCH model, it is necessary to identify ARMA model. Therefore, the Auto regressive moving average (ARMA) Model is identified after making the data stationary. However, there are two methods of identifying the best fit model – Box Jenkins Method and Least information Criteria. ARMA (2,3) model is a model of best fit for the select series. This has been identified on the basis of SIC information criteria.

Table III: Test Results of ARMA (2,3) Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-1.548	0.026	-60.669	0.0000
AR(2)	-0.917	0.025	-36.104	0.0000
MA(1)	1.576	0.032	48.596	0.0000
MA(2)	1.018	0.038	27.070	0.0000
MA(3)	0.092	0.022	4.138	0.0000
R-squared	0.027	Mean dependent var		0.000
Adjusted R-squared	0.026	S.D. dependent var		0.011
S.E. of regression	0.011	Akaike info criterion		-6.185
Sum squared resid	0.270	Schwarz criterion		-6.173
Log likelihood	6960.292	Hannan-Quinn criter.		-6.181
Durbin-Watson stat	2.013			
Inverted AR Roots	-.77-.56i	-.77+.56i		
Inverted MA Roots	-.11	-.73-.57i	-.73+.57i	

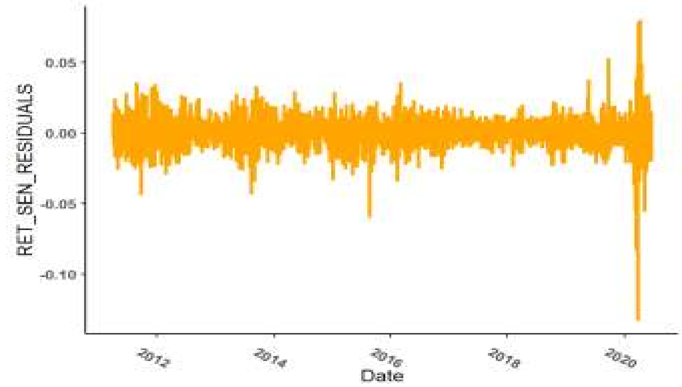
ARMA (2,3) model is chosen as all the variables are significant in this. The conditional mean ARMA(2,3) model equation would be written as follows

$$Y_t = \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \gamma_1 \mu_{t-1} + \gamma_2 \mu_{t-2} + \gamma_3 \mu_{t-3} \quad \text{..., eq-1}$$

Y_t = *Sensex returns in natual logrithm*; $U_t \sim N(0, \sigma^2)$

After this, Ljung-Box test is applied to check the autocorrelation in residuals.

Figure 3: Graph of residuals



Source: The authors'own

The p-values at all lag length in Correlogram were established to be significant which specifies that the series suffer from auto correlation. Heteroskedasticity is then performed using ARCH LM Test, for which suitable lag length is applied using lowest SIC criteria. Lag length of 1 is selected.

Table IV: Outcomes of ARCH LM Test for Heteroskedasticity

F-statistic	58.534	Probability F(1,2246)		0.000
Observed*R-squared	57.098	Prob. Chi-Square(1)		0.000
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000	0.000	9.390	0.000
RESID^2(-1)	0.159	0.021	7.651	0.000

Source: The authors'own

Since the p-value (refer table 4) is less than 0 therefore there is an occurrence of ARCH consequence in residuals. This also shows that there is significant clustering in error term. Therefore, ARCH model is to be identified. The graph of return series (refer Figure 2) indicates that the revenues are not continuous every time and similarly the variance of the return is also not constant. Some shocks make the variance of returns very high; this is followed by sustained period of increase volatility in returns. This leads to 'volatility clustering'.

It means if the volatility of previous period is high, then the volatility of current period as well as for future period may be high. Alternatively, if probability in previous period is low or stable, then it may be a possibility that the volatility may be low in current period and in future period as well until shock is introduced.

ARCH (1) Model

It is always advisable to test the series for any ARCH effect before applying GARCH model to confirm that the data set taken is suitable for applying GARCH.

ARCH (1) model consists of two parts –

1. Equation of Conditional Mean

2. Equation of Conditional Variance

The mean conditional equation is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu^2_{t-1} \qquad \text{..., eq- 2}$$

Table V: ARCH(1) Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.76E-05	2.30E-06	33.74558	0.0000
RESIDUAL(-1)^2	0.357328	0.023154	15.43275	0.0000

Source: The authors'

The significance value of α1 is 0.357328 and corresponding p-value is 0.000. The p-value of resid(-1)^2 is hence significant. This indicates that the squared error period in earlier time period is significant in elucidating the variance of the inaccuracy term in existing time period. After this, residual diagnostics are performed with ARCH LM test and optimum lag length for residual is chosen based on lowest information criteria. Lag length 4 is chosen where the following results are obtained. Q-statistics Correlogram show the presence of auto correlation.

Table VI: Outcomes of Heteroskedasticity ARCH LM Test

F-statistic	27.992	Prob. F(4,2242)		0.0000
Observed*R-squared	106.880	Prob. Chi-Square(4)		0.0000
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.689	0.066	10.417	0.000
WGT_RESID^2(-1)	-0.045	0.021	-2.134	0.033
WGT_RESID^2(-2)	0.149	0.021	7.105	0.000
WGT_RESID^2(-3)	0.105	0.021	5.028	0.000
WGT_RESID^2(-4)	0.103	0.021	4.907	0.000

Source: The authors'

Here, we reject the null hypothesis and we can now proceed to check model of GARCH (1,1).

Model of GARCH (1,1)

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu^2_{t-1} + \beta_1 \sigma^2_{t-1} \qquad \text{..., eq-3}$$

Where, α_0= mean, μ^2_{t-1} is the volatility of a previous time period and is calculated as the lag of squared residuals from mean value and σ^2_{t-1} is the forecast of previous time period.

Therefore, the above equation is able to capture the volatility clustering in stock indices. It is because if there is high volatility in previous time then the forecast will explain a higher volatility in the future period.

Table VII: Test Results for GARCH (1,1) Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.10E-06	5.38E-07	3.907	0.000
RESID(-1)^2	0.086	0.008	10.127	0.000
GARCH(-1)	0.896	0.012	74.933	0.000

Source: The authors'own

Results reported in Table VII indicate entirely the regression coefficients are important at 5% level of significant. Both the ARCH and GARCH parameters are highly significant. This means that any shocks experienced by the provisional variance will be highly insistent as β1.> α1 and the decaying rate is 2%. Also, the coefficient value of both ARCH and GARCH effect together is close to 1, further strengthens the presence of persistence. Further, to check whether ARCH effect is still present, residual diagnostics are performed and Q-statistics indicate that there is no more presence of auto correlation now. ARCH LM test was also performed by choosing the optimum lag length of 1.

Table VIII: Heteroskedasticity Test: ARCH

Observed*R-squared	1.971	Prob. Chi-Square(1)		0.160
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	1.029	0.047	21.984	0.000
WGT_RESID^2(-1)	-0.030	0.021	-1.404	0.161

Source: The authors'own

The observed p-value corresponding to Observed*R-squared is 0.1604, which further specifies that the residuals of the GARCH (1,1) technique (model) do not show ARCH behaviour.

News (bad or good), events, activities such as mergers/ acquisitions, terrorist attacks, pandemic, launch of inventions/new discovery have a bearing on financial markets. A standard ARCH/GARCH model treats news symmetrically. This means the effect of bad news and good news is same on financial assets. However, the impact of bad news and good news vary on assets. For which, GJR Garch Model (TGARCH) is applied. Threshold GARCH model was given by Zakojan(1991a). Therefore it is also known as GJR GARCH. The key objective of TGARCH is to apprehend asymmetries in series with respect to positive and negative shocks.

TGARCH Model

Table 9 reports TGARCH model. Since the p-value of dummy 1 at 5%level of significance is important, it is an indication of sign bias. Also dummy1*garch11resid(-1) and dummy2*garch11resid(-1) too are important at 5% significance level , this represents a strong bias of size.

Table IX: Heteroskedasticity Test: ARCH

Variable	Value of Coefficient	Standard Error	Value of t-Statistic	Probability value
C	-8.32E-05	2.35E-05	-3.543	0.000
DUMMY1	0.000	0.000	4.420	0.000
DUMMY1*GARCH11RESID(-1)	-0.008	0.002	-4.471	0.000
DUMMY2*GARCH11RESID(-1)	0.028	0.002	12.485	0.000

Source: The authors'own

Hence, this study results in a good estimation of GARCH model allowing it for asymmetric volatility. Therefore, GJR GARCH model is applied, which allows for error variance to react according to sign or size of shock it receives.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu^2_{t-1} + \beta_1 \sigma^2_{t-1} + \gamma \mu^2_{t-1} I_{t-1} \qquad \text{..., eq-4}$$

Table X: Test Results of GJR GARCH

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.84E-06	4.30E-07	6.602	0.0000
RESID(-1)^2	-0.003	0.007	-0.388	0.698
RESID(-1)^2*(RESID(-1)<0)	0.164	0.015	10.669	0.000
GARCH(-1)	0.896	0.011	83.753	0.000

Source: The authors'own

? is positive, which is 0.164096, and it is statistically significant which indicates that leverage effect is present. Residual diagnostics have been checked, where p values at all lags is greater than 0.05, hence there was not a problem of auto correlation. The coefficient of Resid(-1)^2(Resid(-1)<0) is confident and p-value is substantial, this means that investors are risk averse and people give different reaction towards bad news and good news. This indicates that there are asymmetries in the news broadcast. Bad news (negative news) has higher impact on volatility of Sensex than any worthy (positive) news.

Conclusion

The present study conducted on SENSEX using ARMA, ARCH and TGARCH indicates that stock markets are driven by sentiments and the subsequent stock prices are affected by previous time periods. This shows the presence of volatility clustering, which is due to persistence. Moreover, the results of TGARCH show that although the market is affected by news, yet negative news led to more fluctuations in market than fluctuations made by positive news. Investors should be more cautious when the bad news arrives, as the volatility increases during such times. Since stock market is an anchorage for funds to many retail and institutional investors, therefore this study will be useful for them in analysing the impact of any event, good or bad, on the market as well as their investment, which will help them in taking informed decisions. They should frame their investment policies by evaluating the current news in the market and predict the future movements so as to receive maximum possible returns and hedge risks. The current research opens up the scope of related studies which can be extended to individual stocks, other financial assets or even other stock markets indices in relation to checking their sensitivity to events happening nationally as well as around the globe.

References

- Akgiray, V. (1989). Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts. Journal of business, 55-80.
- Banerjee, A., & Sarkar, S. (2006). Modeling daily volatility of the Indian stock market using intra-day data. IIMK WPS, 588.
- Bonga, W. G. (2019). Stock Market Volatility Analysis using GARCH Family Models: Evidence from Zimbabwe Stock Exchange. MPRA Paper(94201).
- Brailsford, T. J., & Faff, R. W. (1996). An evaluation of volatility forecasting techniques. Journal of Banking & Finance, 20(3), 419-438.
- Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. Journal of the American statistical association, 74(366a), 427-431.
- Garg, A., & Bodla, B. (2011). Impact of the foreign institutional investments on stock market: Evidence from India. Indian Economic Review, 303-322.
- Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48(5), 1779-1801.
- Goyal, A. (2000). Predictability of stock return volatility from GARCH models. Anderson graduate school of management, UCLA, 110, 90095-91481.
- Gulay, E., & Emec, H. (2018). Comparison of forecasting performances: Does normalization and variance stabilization method beat GARCH (1, 1)-type models? Empirical evidence from the stock markets. Journal of Forecasting, 37(2), 133-150.
- Joshi, P. (2014). Forecasting volatility of Bombay stock exchange. International journal of current research academic review, 2(7), 222-230.
- Lambert, P., & Laurent, S. (2001). Modelling financial time series using GARCH-type models with a skewed student distribution for the innovations. Retrieved from
- Laurent, S., Rombouts, J. V. K., & Violante, F. (2012). On the forecasting accuracy of multivariate GARCH models. Journal of Applied Econometrics, 27(6), 934-955.

- Loudon, G. F., Watt, W. H., & Yadav, P. K. (2000). An empirical analysis of alternative parametric ARCH models. Journal of Applied Econometrics, 15(2), 117-136.
- McMillan, D., Speight, A., & Apgwilym, O. (2000). Forecasting UK stock market volatility. Applied Financial Economics, 10(4), 435-448.
- Ng, H. G., & McAleer, M. (2004). Recursive modelling of symmetric and asymmetric volatility in the presence of extreme observations. International Journal of Forecasting, 20(1), 115-129.
- Pagan, A. R., & Schwert, G. W. (1990). Alternative models for conditional stock volatility. Journal of econometrics, 45(1-2), 267-290.
- Pandey, A. (2005). Volatility models and their performance in Indian capital markets. Vikalpa, 30(2), 27-46.
- Phillips, P. C. B., & Perron, P. (1988). Testing for a unit root in time series regression. Biometrika, 75(2), 335-346.
- Rabemananjara, R., & Zakoian, J.-M. (1993). Threshold ARCH models and asymmetries in volatility. Journal of applied econometrics, 8(1), 31-49.
- Siourounis, G. D. (2002). Modelling volatility and testing for efficiency in emerging capital markets: the case of the Athens stock exchange. Applied Financial Economics, 12(1), 47-55.
- Tripathy, S., & Rahman, A. (2013). Forecasting daily stock volatility using GARCH model: A comparison between BSE and SSE. IUP Journal of Applied Finance(4), 71.
- Vasudevan, R., & Vetrivel, S. (2016). Forecasting stock market volatility using GARCH models: evidence from the Indian stock market. Asian Journal of Research in Social Sciences Humanities, 6(8), 1565-1574.
- Wang, L., Ma, F., Liu, J., & Yang, L. (2020). Forecasting stock price volatility: New evidence from the GARCH-MIDAS model. International Journal of Forecasting, 36(2), 684-694.
- Wilhelmsson, A. (2006). GARCH forecasting performance under different distribution assumptions. Journal of Forecasting, 25(8), 561-578.
- Yu, J. (2002). Forecasting volatility in the New Zealand stock market. Applied Financial Economics, 12(3), 193-202.