Lower Boundary Conditaions and Efficiency Test: Evidence from Indian Options Market

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Abstract

This article attempts to examine the Lower Boundary Conditions (LBCs) on the S&P CNX Nifty index options, the main index of National Stock Exchange (NSE), India. The study covers a period of 5 years that is from 1st April 2012 to 31st March 2017. The violation of the LBCs indicates arbitrage opportunities and it is an indication of the inefficiency of the options market. The study demonstrates frequent violations of LBCs at both the call and put index options. In order to better understand the behaviour of the violations, the study further examines the violations with respect to liquidity and maturity of the options. This has been done with the view that merely violations does not indicate inefficiency of the options market, it is the exploitability of this abnormal profits which gives serious threat to the market efficiency. Further, in order to verify whether the differences in the magnitude of violations are statistically significant hypotheses have been formulated and tested for both the call and put options market. The results of the study indicate that the Indian index options market during the period of study was efficient as most of the abnormal profits were not exploitable due to lack of liquidity.

Keywords: Index options, LBCs, Market Efficiency, Abnormal Profit, Mispricing of Options

Introduction

Options are financial contracts whose values are derived from some underlying asset(s) and they serve as an innovative risk management tool. An Index options, the index itself is the underlying security. Since the introduction of options on the Chicago Board Option Exchange (CBOE) in 1983, index options have become one of the most popular financial instruments. Their simple cash settlement, inexpensive tool for systematic risk management and high leverage capabilities properties adds more attractive to the investment world. They also help in price discovery mechanism and allocating the resources to its most productive uses. However, Indian financial markets took longer time to realize the importance of such a useful financial derivatives instrument. It was on June 2000, National Stock Exchange (NSE) and Bombay Stock Exchange (BSE) commenced trading on index futures followed by index options on June 2001, options on individual stocks on July 2001 and futures on single stock on November 2001. At the embryonic stage of the development of Indian options market there was very low liquidity. But in April 2008 the scenario changed, the volume of trade increased exponentially that is 55,366,038 number of

index options contracts traded in 2007-08 jump to 212,088,444 number of index options contracts traded in 2008-2009. Index options are the most traded instrument in the derivative market segment of India (more than 63 per cent of the total derivative trading recorded under index options in 2010-11). Given all these important functions the efficiency test of options market is important to academicians, practitioners as well as the regulators. A number of studies have been done to test the efficiency of options market around the globe. But most of the studies are done at well-developed markets like American and Europe where options are employed at its maximum utility. In this study, the efficiency of the Indian index options market has been investigating by applying ex-post analysis to the daily closing prices observed for both the call and put index options. The LBCs for the options prices has been used as a tool to gauge the efficiency of the Indian index options market. The violation of the LBCs implies that the options are underpriced and hence there is an arbitrage opportunity. The violations signals observed from the test procedures have been classified as per time-to-maturity and liquidity to facilitate a meaningful explanation in assessing the efficiency of the market. This has been done with the view that merely violations does not indicate inefficiency of the options market, it is the exploitability of this abnormal profits which gives serious threat to the market efficiency.

The study is divided into five main sections. The next section describes the review of literature and theoretical framework of the study. The data for the study are described in the third section. The fourth section presents the analysis and results of the study and the last section present the conclusion and policy implications of the study.

Review of Literature and Theoretical Framework

The Lower Boundary Conditions (LBCs) is a well-known technique to gauge the efficiency of the options market first proposed by Merton (1973) and further extended by Galai (1978). It denotes the minimum price of the options at a given point of time; its violation indicates arbitrage opportunities. Around the globe, a number of research studies have been carried out at different options markets using the LBCs to assess the efficiency of the markets including the first one by Galai (1978) on the Chicago Board Option Exchange (CBOE). The other studies which tried to diagnose the options market efficiency based on the violation of the LBCs include Bhattacharya (1983) on the CBOE, Chance (1988) on the S&P 100 index options market for the United States (US), Puttonen (1993) on the Finnish Option Market (FOM), Ackert and Tian (2000, 2001) on S&P 500 index options market for the US, Mittnik and Rieken (2000) on the German stock index options DAX. Halpern and Turnbull (1985) on Toronto options market, Canadian.

In Indian context only a handful of studies have done for testing the Indian index options market applying LBCs. They are as follows Dixit et al. (2009, 2011), Mohanti and Priyan (2013), but all these studies are either done at the initial development stage of the Indian options market or when the microstructure of the Indian Exchanges are under the strict rule of the Regulator, namely restriction of short selling, brokerage charges. But currently there are notable changes in the microstructure of the exchanges in Indian some of them are relaxation of shorting, settlement on (T+1) or (T+0) day (here T stand for trading day), brokerage charge on premium only and in mean time the volume of options trading have increase tremendously. During the calendar year 2016, NSE ranked first in the world in respect of the contracts traded in Index options (FIA). So testing the efficiency of the Indian options market is in need as this market play important role in the development of the economy. As mentions earlier that options help in risk management, price discovery mechanism and proper allocation of resources.

The present study is confined to Indian context, S&P CNX Nifty index options market, they are European types (which can be exercised only at maturity) and settled by cash. The LBCs for the call and put options are given in equations (1) and (2) respectively; these conditions have to be satisfied to avoid arbitrage.

$$C_t \geq \max (0, I_t - K e^{-r(T-t)}) \quad (1)$$

$$P_t \geq \max (0, K e^{-r(T-t)} - I_t) \quad (2)$$

In equations (1) and (2), Ct is the market price of the call options at time t, Pt is the market price of the put options at time t, It is the level of the underlying index at time t, K is the strike price of the options contract, T is the expiration time of the options contract, r is the continuously compounded annual risk-free rate of return and (T - t) is the time to maturity of the options at time t (measured in years).

Here equations (1) and (2) does not incorporate the effect of dividends, assuming the underlying index pays continuously compounded annual dividend yield (δ) equations (1) and (2) are transformed to equations (3) and (4) respectively. This transformation is in line with Chance (1988) and Dixit et al. (2009).

$$C_t \geq \max (0, I_t e^{-\delta(T-t)} - K e^{-r(T-t)}) \quad (3)$$

$$P_t \geq \max (0, K e^{-r(T-t)} - I_t e^{-\delta(T-t)}) \quad (4)$$

Here the underlying asset is an index it will be difficult to buy or sell the equal proportion to shares composing the index. To overcome this difficulty futures contract on the same index have been employed in the study as futures can be easily buy or sell. In India both the index options and futures are traded on the same exchange. They have the same contract tenure (expiration day), same contract size, same trading timing. Because of all these similarity futures will be best proxy of the index. This conversion is in line

with the study done by Dixit et al. (2011).

The equations (3) and (4) are again transformed to equations (5) and (6) as follows:

$$C_t \geq \max \{0, (F_t - K)\, e^{-r(T-t)}\} \quad (5)$$

$$P_t \geq \max \{0, (K - F_t)\, e^{-r(T-t)}\} \quad (6)$$

In the equations (5) and (6), Ft represents the futures price of the index at time t.

From the above equations (5) and (6), the testable forms of the LBCs are given by equations (7) and (8) as follows:

$$\varepsilon^c_t = \{(F_t - K)\, e^{-r(T-t)}\} - C_t \quad (7)$$

$$\varepsilon^p_t = \{(K - F_t)\, e^{-r(T-t)}\} - P_t \quad (8)$$

In the above equations (7) and (8), Ɛct and Ɛpt are the absolute amount of abnormal profits from the call and put options respectively. The violation of the LBCs are recorded, if Ɛct > 0 and Ɛpt > 0 in the case of the call and put options respectively.

Although the equations (7) and (8) give a positive result, it only indicates market inefficiency but it should not be treated as a conclusive remark on the efficiency of the market, as the study procedure is ex-post in nature. And in mean time equations (7) and (8) assumed no transaction costs. It may be noted that there is always a chance that the arbitrage opportunities suggested by these equations may disappear in the presence of transaction costs, Galai (1978), Bhattacharya (1983).

The Data

The data employed for the study can be classified into three categories. First, the options contract collected from the NSE, India website. It consists of daily closing prices of options, strike prices, deal dates, maturity dates and the number of the call and put index options. In order to minimise the bias associated with non-synchronous trading, only liquid options quotations (contracts that have at least one contract traded in a day) are considered for the study. Although three month contracts Near (current month), Next (one month next to current month) and Far (two months next to current month) and others long-term options contracts are available at a point of time in the Indian options market only Near month options contract are consider for the study due to lack of liquidity in the other options contracts, as reported by Dixit et al. (2009, 2011), Bhattacharya (1983), Mohanti and Priyan (2013). The second data sets, the underlying asset, Nifty index futures are also collected from the NSE, India website. It includes the daily closing price of Nifty index futures. The third data set consists of monthly average yield on the 91-day Treasury-bills collected from the Reserved Bank of India (RBI) website. The data on Treasury-bills have also been converted into a continuously compounded annual rate of return. All these three categories of data have been collected from 1 April 2012 to 31 March 2017 from the above-stated sources.

Analyses and Empirical results

The ex-post analysis of the call and put index options have been carried out on the basis of equations (7) and (8) from 1 April 2012 to 31 March 2017. The LBCs has been conducted for 101087 daily liquid index options quotes, out of which 50641 are for the call index options and 50446 are for the put index options. Table 1, reported a total of 14084 and 11278 mispricing signals for the call and put index options respectively. The analysis clearly reveals that the violations are higher for the call index options (about 27.81 per cent of the total observed call index options) than that of the put index options (about 22.36 per cent of the total observed put index options). These violations have been increased as compared with the previous study done at the embryonic stage of the Indian index options market by Dixit et al. (2011), about 8.92 per cent for the call and 5.16 per cent violation for the put index options.

Another important observation in Table 1 is that majority of the violations about 71.12 per cent for the call and about 65.75 per cent for the put index options belong to the thinly traded options level, which lack exploitability opportunity due to high transaction cost (high bid-ask spread). This observation is in line with the observations made at the embryonic stage of the options market in India by Dixit et al. (2009, 2011). And as the liquidity increases violations decreases this is a good sign for the efficiency of the index options market.

Table 1: Violations of the lower boundary conditions and liquidity levels
(number of observations)

Particulars	Call options	Put options
Total number of observations analyzed	50641	50446
Total number of violations observed	14084 (27.81)	11278 (22.36)
Violations relating to the three specified levels of liquidity		
(a) Thinly traded options (less than 500 contracts traded per day)	10017 (71.12)	7451 (65.75)
(b) Moderately traded options (more than 500 but less than 2000 contracts traded per day)	2589 (18.38)	2266 (20.09)

(c) Highly traded options (more than 2000 contracts traded per day)	1478 (10.50)	1597 (14.16)
Total	**14084 (100)**	**11278 (100)**

Source: Author's calculations.
Note: Figures in parentheses indicate percentage

In order to extract more information about the violation of LBCs, the violations have been again classified as per the maturity of the index options contracts (days left for the options expiration). From Table 2, it has been observed that about 36.56 per cent of the total violated call index options and about 33.57 per cent of the total violated put index options are concentrated at the expiration date, that is, 0-7 days left for expiration this observation is in line with Dixit et al. (2009, 2011), Bhattacharya (1983) and Chance (1988). And the remaining violated options for both the call and put index options are scattered around the other levels of day-to-maturity as observed in Table 2.

Table 2: Violations of the lower boundary conditions and maturity levels (number of observations)

Maturity	**Call options**		**Put options**	
(in day)	Number	percentage	Number	percentage
0–7	5149	36.56	3786	33.57
8–14	3212	22.81	2596	23.02
15–21	2766	19.64	2346	20.80
22–30	2957	21.00	2550	22.61
Total	**14084**	**100**	**11278**	**100**

Source: Author's calculations.

To obtain more meaningful information about these violations, cross tables across the specified level of maturity and liquidity for both the call and put index options are shown in Tables 3 and 4 respectively. From Tables 3 and 4, it is observed that for both the call and put index options most of the observed violated highly liquid options (Table 1) belong to the options contracts which are going to get matured (Table 2). So in spite of having high liquidity in the market, extracting abnormal profit is quite impossible as the options are going to get expired and there will be more sellers comparing to the buyers, as all the arbitragers will close down their open positions. It is also observed from Tables 3 and 4 that majority of the violations are more at the put index options as compared to the call index options at the highly traded level which have 8-14day, 15-21 days and 22-30 days left for the maturity. This reflects that the opportunity of extracting the abnormal profits are more in the put index options market comparing to the call index options market, as the transaction cost will be low due to low bid-ask spread and more liquidity.

Table 3: Violations of lower boundary conditions for the call index options cross-tabulated across specified levels of liquidity and maturity (number of observations).

	Liquidity		
Maturity	Thinly traded	Moderately traded	Highly traded
0–7 days	2557	1340	1252
8–14 days	2554	530	128
15–21 days	2383	342	41
22–30 days	2523	377	57
Total	**1017**	**2589**	**1478**

Source: Author's calculations.

Table 4: Violations of lower boundary conditions for the put index options cross-tabulated across specified levels of liquidity and maturity (number of observations)

	Liquidity		
Maturity	Thinly traded	Moderately traded	Highly traded
0–7 days	1888	829	1069
8–14 days	1827	513	256
15–21 days	1763	442	141
22–30 days	1937	482	131
Total	**7415**	**2266**	**1597**

Source: Author's calculations.

The Tables 5 and 6 shows the descriptive statistics of the absolute amount of the violations across the three specified levels of liquidity and the four specified levels of the maturity respectively for both the call and put index options. From Table 5, it can be observed that for both the call and put index options the numbers of violations have been reduced as the liquidity increases. But in the case of four specified levels of maturity it is observed that for both the call and put index options the numbers of violations reduces as the contracts are going to get expired (Table 6). All these signs are good for the market as they all indicate options market efficient.

Table 5: Descriptive statistics of the absolute amount of the violations across the three specified levels of liquidity (figures in Rupee per contract)

Liquidity	Mean		Std. Deviation		Minimum		Maximum	
	Call	Put	Call	Put	Call	Put	Call	Put
Thinly	12.12	16.58	19.60	19.34	0.00	0.00	503.06	231.73
Moderately	4.50	8.80	4.05	9.21	0.00	0.00	58	70.23
Highly	3.58	5.96	4.43	5.98	0.00	0.01	103.04	43.78
Total	9.82	13.51	17.07	16.93	0.00	0.00	503.06	231.73

Source: Author's calculations.

Table 6: Descriptive statistics of the absolute amount of the violations across the four specified levels of maturity (figures in Rupee per contract)

Maturity (in days)	Mean		Std. Deviation		Minimum		Maximum	
	Call	Put	Call	Put	Call	Put	Call	Put
0–7	7.77	10.22	15.09	15.64	0.00	0.01	438.20	231.73
8–14	9.86	13.41	18.15	16.17	0.00	0.01	503.06	225.80
15–21	10.80	14.91	14.23	16.24	0.00	0.00	190.62	147.37
22–30	12.47	17.22	20.71	19.10	0.00	0.00	324.69	203.19
Total	9.82	13.51	17.07	16.93	0.00	0.00	503.06	231.73

Source: Author's calculations.

Following the descriptive statistics of the absolute amount of violations, further analysis has been carried out to identify whether there exists a difference across the three specified levels of liquidity and the four specified levels of days left for maturity as specified in Tables 1 and 2. One-way analysis of variance (ANOVA) is a good choice for this analysis. This statistical test will facilitated analysis the behaviour of the absolute amount of violations in relation to the different specified levels of liquidity and different specified levels of the days left to maturity. The analysis will provides better insights into the exploitability of abnormal profits indicated by the observed mispricing signals. But before applying ANOVA on the samples data, the assumptions of normality have been tested using one-sample Kolmogorov-Smirnov statistics. The test results show violations of normality as summarised at Table 7. ANOVA can't be applied but its analogous non-parametric statistics which doesn't require the data to follow any specified distribution; Kruskal-Wallish (H-statistics) test have been employed.

Table 7: Summary of One-Sample Kolmogorov-Smirnov statistics to Assess Normality

Variable		**Call Options**	**Put Options**
Number of Observations		14084	11278
Normal Parameters (a, b)	Mean	9.82	13.51
	Std. Deviation	17.07	16.93
Most Extreme Differences	Absolute	0.282	0.212
	Positive	0.251	0.192
	Negative	-0.282	-0.212
Kolmogorov-Smirnov Z		33.52	22.56
ASymp. Sig. (2-tailed)		**0.000**	**0.000**

a. Test distribution is Normal. b. Calculated from data.

In addition to Kruskal-Wallish (H-Statistics) test, Dunn's multiple comparison tests has been employed for the post-hoc analysis of all possible pairs in the analysis. The results of H statistics and Dunn's test for the differences across the specified levels of liquidity and maturity are summarized in tables 8, 9 (a) and (b), 10, 11 (a) and (b) respectively. All the test results show statistical significant at a significant level of 5 per cent.

In operational term, the results show that the absolute amount of violations for the thinly traded call and put index options are significantly different from those for the moderately traded and highly traded call and put index options. In the case of the days left for maturity, for both the call and put index options the absolute amount of the violations for all the possible pairs are significantly different from each others. From this finding and those given in Tables 5 and 6 show that the majority of the violations of the highly traded put index options are more consistent, as their standard deviation of mean is lower than in the case of the call index options and are thus more consistently exploitable compared to those in the case of the call index options.

Table 8 (a): Kruskal-Wallis (H-statistics) test for the difference among the violations across the Specified level of liquidity for call index options

Liquidity	Ranks		Test Statistic (a, b)		
	N	Mean Rank	Chi-Square	df	Sig.
Thinly	10017	7966.50			
Moderately	2589	5093.36	1837.529*	2	0.000
Highly	1478	4194.51			
Total	14084				

a. Kruskal Wallis Test b. Grouping Variable: Liquidity
Note: * indicate statistical significance at 5 per cent level

Table 8 (b): Dunn's test for multiple comparisons amongst the specified levels of liquidity for call index options

Dunn's Multiple comparison Test	Test Statistic	Sig.
Highly Vs Moderately	898.85*	0.000
Highly Vs Thinly	3771.99*	0.000
Moderately Vs Thinly	2873.14*	0.000

Note: * indicate statistical significance at 5 per cent level

Table 9 (a): Kruskal-Wallis (H-statistics) test for the difference among the violations across the Specified level of liquidity for put index options

Liquidity	Ranks		Test Statistic (a, b)		
	N	Mean Rank	Chi-Square	df	Sig
Thinly	7415	6359.40			
Moderately	2266	4667.72	1145.32*	2	0.000
Highly	1597	3675.83			
Total	11278				

a. Kruskal Wallis Test b. Grouping Variable: Liquidity
Note: * indicate statistical significance at 5 per cent level

Table 9 (b): Dunn's test for multiple comparisons amongst the specified levels of liquidity for put index options

Dunn's Multiple comparison Test	Test Statistic	Sig.
Highly Vs Moderately	991.89*	0.000
Highly Vs Thinly	2683.57*	0.000
Moderately Vs Thinly	1691.68*	0.000

Note: * indicate statistical significance at 5 per cent level

Table 10 (a): Kruskal-Wallis (H-statistics) test for the difference among the violations across the Specified level of maturity for call index options

Maturity (in days)	Ranks		Test Statistic (a, b)		
	N	Mean Rank	Chi-Square	df	Sig
0-7	5149	6000.66			
8-14	3212	6982.85			
15-21	2766	7706.68	695.51*	3	0.000
22-30	2957	8300.15			
Total	14084				

a. Kruskal Wallis Test b. Grouping Variable: Maturity
Note: * indicate statistical significance at 5 per cent level

Table 10 (b): Dunn's test for multiple comparisons amongst the specified levels of maturity for call index options

Dunn's Multiple comparison Test		
Maturity (in days)	Test Statistic	Sig.
0-7 Vs 8-14	-981.19*	0.000
0-7 Vs 15-21	-1706.01*	0.000
0-7 Vs 22-30	-2299.49*	0.000
8-14 Vs 15-21	-723.820*	0.000
8-14 Vs 22-30	-1317.30*	0.000
15-21 Vs 22-30	-593.48*	0.000

Note: * indicate statistical significance at 5 per cent level

Table 11(a): Kruskal-Wallis (H-statistics) test for the difference among the violations across the Specified level of maturity for put index options

Maturity (in days)	Ranks		Test Statistic (a, b)		
	N	Mean Rank	Chi-Square	df	Sig
0-7	3786	4719.72			
8-14	2596	5699.20			
15-21	2346	6092.87	538.09*	3	0.000
22-30	2550	6527.23			
Total	11278				

a. Kruskal Wallis Test b. Grouping Variable: Maturity
Note: * indicate statistical significance at 5 per cent level

Table 11(b): Dunn's test for multiple comparisons amongst the specified levels of maturity for put index options

Dunn's Multiple comparison Test Maturity (in days)	Test Statistic	Sig.
0-7 Vs 8-14	-979.48*	0.000
0-7 Vs 15-21	-1373.15*	0.000
0-7 Vs 22-30	-1807.51*	0.000
8-14 Vs 15-21	-393.66*	0.000
8-14 Vs 22-30	-828.02*	0.000
15-21 Vs 22-30	-434.36*	0.000

Note: * indicate statistical significance at 5 per cent level

Conclusion and Policies Implications

This study reveals that the violations of LBCs are more frequent in the period of the study comparing to the initial development stage of the index options market in India. Both the options are not correctly priced. This suggests that both the call and put index options are underpriced. The finding that the index options are underpriced is consistent with the previous studies done by Varma (2002), Dixit et al. (2009, 2011) and Vipul (2008). But in spite of more frequent violations of LBCs prevailing in the Indian index options market, exploitability of the arbitrage profit is quite possible as most of the violations are at thinly traded level and options which are going to get expired. And the study was also conducted without transaction costs as transaction costs are difficult to estimate. It not only differ for different participants in the market namely retail investors, institutional investors, full brokerage firms, discount brokerage firms. It also differs by the types of trading strategies employed. Further scope of the study is to incorporate transaction costs while testing the LBCs. From all the tests and analysis of the study, the study concludes that the Indian index options market during the periods of the study was efficient enough as most of the abnormal profits were not exploitable as most of the violations are at thinly traded level and options which are going to get expired that is lacks of liquidity.

The finding of this study will be useful to all types of investors, portfolio managers, traders who trade in derivatives (especially to the emerging derivatives market) and it will also be useful for the stock exchanges, policy makers, regulators and other concern authorities nationally and internationally who are involved in this market.

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